

## Limitations of $R_s-R_p$ technique for evaluating $n$ and $k$ 953

The above graphs clearly demonstrate that the  $R_s-R_p$  method, though simple, cannot be applied for high reflecting films. Since in this films an error less than 5% is tolerable, the  $R_s$  of the sample should generally be low ( $< 0.5$ ), whilst measurement should be accurate enough such that  $(R_p-R_s^2) < \pm 0.002$  which is not beyond experimental limitations. Hence this technique is suitable for the estimation of  $n$  and  $k$  only when both  $R$  and  $(R_p-R_s^2)$  are low. This conclusion has been verified by us for many vacuum deposited absorbing and semiabsorbing films.

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## Interaction of bounded electromagnetic beams with moving homogeneous dielectric media\*

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With the recent and rapid progress in space exploration, the electromagnetic field problems in moving media have become quite important. Most of the studies (Lee & Papas 1964, Pyati 1967, Yeh 1965, 1966, Yeh & Casey 1966) made to date are confined to infinitely extended plane electromagnetic (EM) waves. However, under experimental and practical conditions, we are always concerned with bounded EM beams rather than infinitely extended plane EM waves, therefore it is desirable to examine the peculiarities that arise in reflection of bounded beams from moving homogeneous, isotropic dielectric medium. The method of investigation is based on the expansion of bounded beams into infinite set of plane waves by Fourier analysis techniques, whereas the reflection of plane EM wave has been widely studied (Yeh 1965, 1966; Yeh & Casey 1966).

The formulation of the problem under consideration essentially reduces to the study of interaction of bounded EM beams in a frame ( $K'$ ), which is at rest with respect to the moving medium then transforming back into the observer's

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frame ( $K$ ), by using covariance of Maxwell's equations, phase invariance of plane EM waves and the Lorentz transformations and thereafter interpreting the results in terms of the velocity of the moving media; direction of motion and angle of incidence.

Following Brekhovskikh (1966), displacement suffered by the beam on total reflection from dielectric half-space and dielectric slab of thickness  $d$ , in the reference frame  $K'$  (with electric vector polarized perpendicular to the plane of incidence) is given by

$$\Delta'_{DHS} = \frac{p_x'/p_z'}{[k_x'^2 - k^2(\epsilon_1/\epsilon_0)]^{\frac{1}{2}}} \quad \dots (1)$$

and

$$\Delta_{DS'} = \frac{2k_x'[k_z'\{\coth(Pd) \cdot Q/P + 4P \coth(Pd) + dQ(1 - \coth^2(Pd))\} - PQ \coth(Pd)]/k_z'}{Q^2 + 4k_z'^2 P^2 \coth^2(Pd)} \quad \dots (2)$$

where

$$P^2 = [k_x'^2 - \omega'^2 \mu_0 \epsilon_1],$$

and

$$Q = [k_z'^2 - k_x'^2 + \omega'^2 \mu_0 \epsilon_1].$$

Case (a)

Let us consider the case in which the medium is moving in  $x$ -direction with a relative velocity  $v_x = c\beta_x$ , then using the covariance of Maxwell's equations, the phase invariance principle of plane waves and the Lorentz transformations (Yeh 1965, 1966; Yeh & Casey 1966), eqs (1) and (2) in the observer's frame are given by [in the approximation  $\beta_x^2 \ll 1$ ]:

$$\Delta_{DHSx} = \frac{(2c/\omega)(\sin \theta_0 - \beta_x)/\cos \theta_0}{[(\sin \theta_0 - \beta_x)^2 - (1 - \beta_x \sin \theta_0)^2(\epsilon_1/c_0)]^{\frac{1}{2}}} \quad \dots (3)$$

and

$$\Delta_{DSx} = \frac{2(\omega/c)(\sin \theta_0 - \beta_x)[(\omega/c) \cos \theta_0 \{Q_1 \coth(P_1 d)/P_1 + dQ_1(1 - \coth^2(P_1 d)) + 4P_1 \coth(P_1 d)\} - \frac{P_1 Q_1 \coth(P_1 d)}{(\omega/c) \cos \theta_0}]}{Q_1^2 + 4(\omega/c)^2 \cos^2 \theta_0 P_1^2 \coth^2(P_1 d)} \quad \dots (4)$$

where

$$P_1^2 = (\omega/c)^2[(\sin \theta_0 - \beta_x)^2 - (1 - \beta_x \sin \theta_0)^2(\epsilon_1/\epsilon_0)],$$

and

$$Q_1 = (\omega/c)^2[\cos^2 \theta_0 - (\sin \theta_0 - \beta_x)^2 + (1 - \beta_x \sin \theta_0)^2(c_1/\epsilon)].$$

Case (b)

Let us consider the case in which the rare medium is moving in  $z$ -direction with a relative velocity  $v_z = c\beta_z$  then using the covariance of Maxwell's equations

the phase invariance principle of plane wave and the Lorentz transformations eqs (1) and (2) in the observer's frame are given by [in the approximation  $\beta_z^2 \ll 1$ ]:

$$\Delta_{DHSZ} = \frac{2(c/\omega) \sin \theta_0 (\cos \theta - \beta_z)}{|\sin^2 \theta_0 - (1 - \beta_z \cos \theta_0)^2 (c_1/c)|^{1/2}} \quad (5)$$

and

$$\Delta_{DSZ} = - \frac{2(\omega/c) \sin \theta_0 \left[ (\omega/c)(\cos \theta_0 - \beta_z) \{ Q_2 \coth(P_2 d)/P_2 + Q_2 d(1 - \coth^2(P_2 d)) + 4P_2 \coth(P_2 d) \} - \frac{P_2 Q_2 \coth(P_2 d)}{(\omega/c)(\cos \theta_0 - \beta_z)} \right]}{Q_2^2 + 4(\omega/c)^2 (\cos \theta_0 - \beta_z)^2 P_2^2 \coth^2(P_2 d)} \quad \dots \quad (6)$$

where

$$P_2^2 = (\omega/c)^2 [\sin^2 \theta_0 - (1 - \beta_z \cos \theta_0)^2 (c_1/c_0)],$$

and

$$Q_2 = (\omega/c)^2 [(\cos \theta_0 - \beta_z)^2 + (1 - \beta_z \sin \theta_0)^2 (c_1/c_0) - \sin^2 \theta_0]$$

From eqs. (3-4) and (5-6) we see that the interaction of bounded EM beams with moving homogeneous isotropic media under the condition of total internal reflection, leads to the skipping of the beam at the surface, skipping being complicated function of the velocity of the medium, the direction of motion and angle of incidence

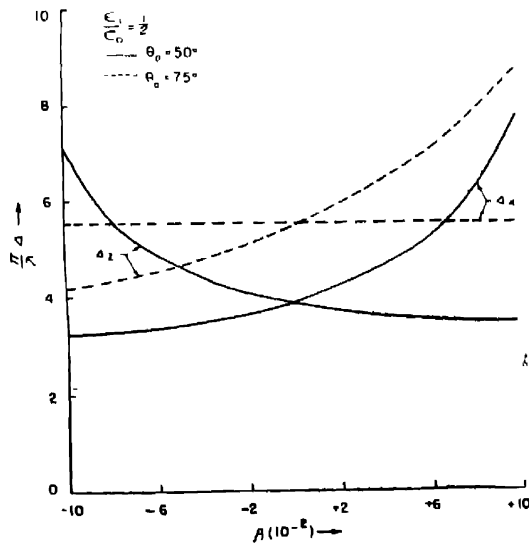


Fig. 1. Variation of displacement  $(\pi/\lambda)\Delta$  as a function of velocity for various angles of incidence with  $c_1/c_0 = 1/2$ .

Some numerical results have been presented to show the displacement suffered by the beam on reflection from the dielectric half-space. Figure 1 shows the variation of displacement as a function of velocity for various angles of incidence with  $\epsilon_1/\epsilon_0 = 1/2$ . Figure 2 shows the variation of displacement with velocity for the situation when  $\epsilon_1/\epsilon_0 = 1/2$  and  $\epsilon_2/\epsilon_0 = 1/3$  with  $\theta_0 = 75^\circ$

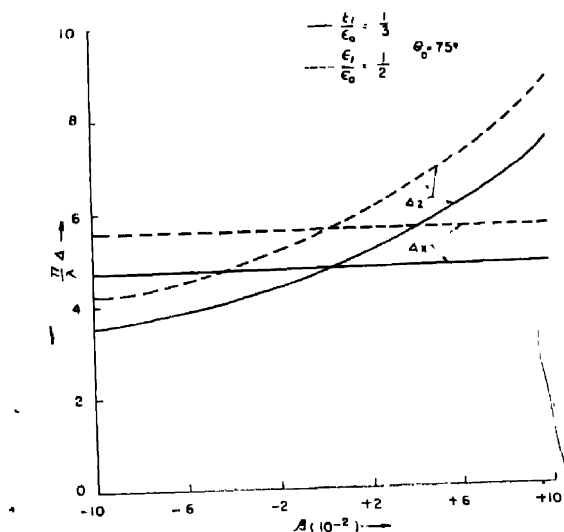


Fig. 2. Variation of displacement  $(\pi/\lambda)\Delta$  with velocity for the situation when  $\epsilon_1/\epsilon_0 = 1/2$  and  $\epsilon_2/\epsilon_0 = 1/3$  with  $\theta_0 = 75^\circ$ .

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